First Semester M.Tech. Degree Examination, May/June 2010
Applied Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Define: i) error ii) relative error iii) round-off error iv) inherent error, and v) truncation error. Add the numbers 83.72 and 1.529 in a decimal computer, with a fixed word-length 4. Find the absolute and relative errors involved. **(10 Marks)**

b. Write an algorithm for Gauss elimination method of solving a system of linear algebraic equations. **(10 Marks)**

2 a. Solve the following system of equations by using the LU decomposition method:

$$x + y + z = 1$$
$$3x + y - 3z = 5$$
$$x - 2y - 5z = 10$$

(10 Marks)

b. Use the Gauss-Seidel method to solve the system:

$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$

Carry out five iterations. **(10 Marks)**

3 a. Using the Jacobi method, find all the Eigen values and the corresponding Eigen vectors of the matrix $A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$ **(10 Marks)**

b. Find the numerically largest Eigen value and the corresponding Eigen vector of the matrix, using power method.

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

(10 Marks)

4 a. Given

x :	1.0	1.2	1.4	1.6	1.8	2.0
y :	2.72	3.32	4.06	4.96	6.05	7.39

find the dy/dx and d^2y/dx^2 at $x = 1.2$. **(10 Marks)**

b. Find the Jacobian matrix for the system of equations :

$f_1(x, y) = x^2 + y^2 - x = 0$; $f_2(x, y) = x^2 - y^2 - y = 0$

at the point (1, 1), using the methods

$$\left(\frac{\partial f}{\partial x}\right)_{(x_i, y_J)} = \frac{f_{i+1,J} - f_{i-1,J}}{2h} \quad ; \quad \left(\frac{\partial f}{\partial y}\right)_{(x_i, y_J)} = \frac{f_{i,J+1} - f_{i,J-1}}{2k},$$ with $h = k = 1$. **(10 Marks)**

5 a. Write an algorithm for Simpson's rule. **(10 Marks)**

b. Estimate $\int_0^{0.5}\int_0^{0.5} \frac{\sin xy}{1+xy} dxdy$, using Simpson's rule for double integrals with both step sizes equal to 0.25. **(10 Marks)**

6 a. Write an algorithm for Euler's method to solve the initial value problem. **(10 Marks)**

b. Use Milne's predictor-corrector method to find y, when x = 0.8, given $dy/dx = x - y^2$, y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Apply Corrector's formula twice. **(10 Marks)**

7 a. Find the solution of the boundary value problem $y'' = y + x$, $x \in [0, 1]$, with y(0) = 0, y(1) = 0, using the shooting method. Use the Runge-Kutta method of second order to solve the initial value problems with h = 0.2. **(10 Marks)**

b. Solve the boundary value problem using finite difference method:
dy/dx = x + y with y(0) = y(1) = 0. **(10 Marks)**

8 a. Solve the two-dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at the interior pivotal points of the square region shown in the following Fig.Q8(a).

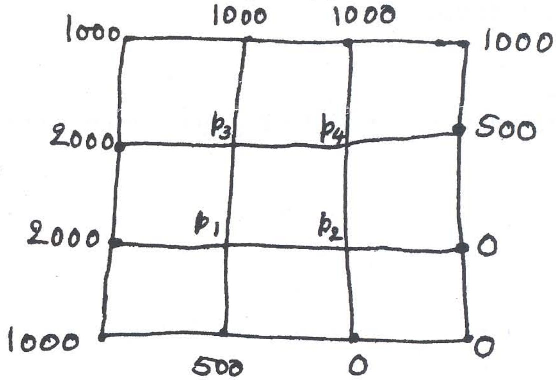

Fig.Q8(a) **(10 Marks)**

b. Employ the Crank-Nicolson method to solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the initial and boundary conditions $u(x, 0) = \sin\pi x$, $0 \le x \le 1$; $u(0, t) = u(1, t) = 0$ for h = 1/3 and k = 1/36. Integrate upto two time levels. **(10 Marks)**